**3B**

**5.solution: assume that (e1,e2,e3,e4) is a basis of R4, we define T∈L(R4,R4) that: Te1=Te2=0,Te3=e1,Te4=e2.note that nullT=rangeT=span(e1,e2), hense the nullT=rangeT.**

**6.see the solution**

**10.conclusion: suppose (v1,...,vn) spans V and T∈L(V,W), prove that (Tv1,...,Tvn)spans rangeT.**

**Similar conclusion: suppose (v1,...,vn) spans V and T∈L(V,W) is surjective, prove that (Tv1,...,Tvn)spans W.**

**point: prove that rangeT⊂span(Tv1,...,Tvn) and rangeT⊃span(Tv1,...,Tvn)**

**12.solution: there exsists a subspace U of V that V = U⊕nullT. From the defination of direct sum we have U∩nullT={0}.below, we should proof that rangeT={Tu:u∈U}. obviously, {Tu:u∈U}⊂rangeT. To proof the other hand, suppose that v∈V. There exists u`∈U and w∈nullT such that v=u`+w. Applying T to both sides so that Tv=T(u`+w)=Tu`+Tw=Tu` which means Tv∈{Tu:u∈U}. cause Tv is an arbitrary vector in rangeT(since v is an arbitrary vector in V)， rangeT⊂{Tu:u∈U}.(here in my own opinion,we should proof that Tv can be linear represented by a vector in {Tu:u∈U} so that Tv is a vector in {Tu:u∈U}, and Tv must be a arbitrary vector in rangeT) hence rangeT={Tu:u∈U}.**

**13.Solution: Since T∈L(F4,F2), we know that ((1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)) is a basis of V. since nullT is a subspace of v, (x1,x2,x3,x4) can be rewritten below: (x1,x2,x3,x4)=x2(5,1,0,0)+x4(0,0,7,1). Hence ((5,1,0,0),(0,0,7,1)) is a basis of nullT in the condition of x2 and x4 are arbitrary. Thus dimnullT=2 and dimrangeT=dimV-dimnullT=2 which means rangeT=F2. Thus rangeT=W and T is surjective as desire.**

**15. Prove that there does not exist a linear map from F5 to F2 whose null space equals to**